UNIT 5:

LAWS OF PLANETARY MOTION

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TEACHER'S GUIDE

In this unit we have concentrated all the laws of motion and we intend that the students can obtain physical data by applying these laws, either with the data provided or with those that the students themselves can get with the measurements they make on the astronomical images.

Due to the level of physics required and explained in this unit, it is designed for high school students. The use of a scientific calculator and the use of exponential functions will be necessary.

Next, we will show the results of each activity.

ACTIVITY 1

CALCULATION OF THE MASS OF THE SUN

The following is the verification of Kepler's 3rd law, where P^2/a^3 is constant.

Planet	Period (years)	Semi-major axis (U.A)	P^2/a^3
Mercury	0.24	0.387	0.99378
Venus	0.62	0.723	1.01711
Earth	1.00	1.000	1.00000
Mars	1.88	1.524	0.99853
Jupiter	11.86	5.203	0.99864
Saturn	29.46	9.539	0.99990
Uranus	84.01	19.182	0.99995
Neptune	164.80	30.058	1.00008

Using our planet's orbital period and average distance from the Sun as units of measurement, we find that the ratio remains practically constant and equal to 1.

- Data provided in the unit
- Data obtained by performing the necessary calculations

We will now explain how to derive the formula for calculating the mass of the Sun or any massive body over which it orbits.

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

Suppose we want to calculate the mass ***M*** of a body which we know is orbited by another body of smaller mass ***m***, whose trajectory is circular, and with a mean orbital radius **a**, going through a complete orbit in a period of time ***P***.

For the smaller body to remain in orbit, the gravitational force that attracts it to the body of greater mass must be compensated by the centrifugal force due to the circular motion, which goes in the opposite direction. Therefore, we obtain:

$$F_g = F_c \quad\rightarrow\quad G\frac{Mm}{a^2} = m\frac{v^2}{a} \quad\rightarrow\quad G\frac{Mm}{a^2} = m\frac{\left(\frac{2\pi a}{P}\right)^2}{a} \quad\rightarrow\quad G\frac{Mm}{a^2} = \frac{m 4\pi^2 a^2}{P^2 a}$$

And so:

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

where ***G*** is the universal gravitational constant and is 6.67x10^{-11} $N\ m^2\ kg^{-2}$.

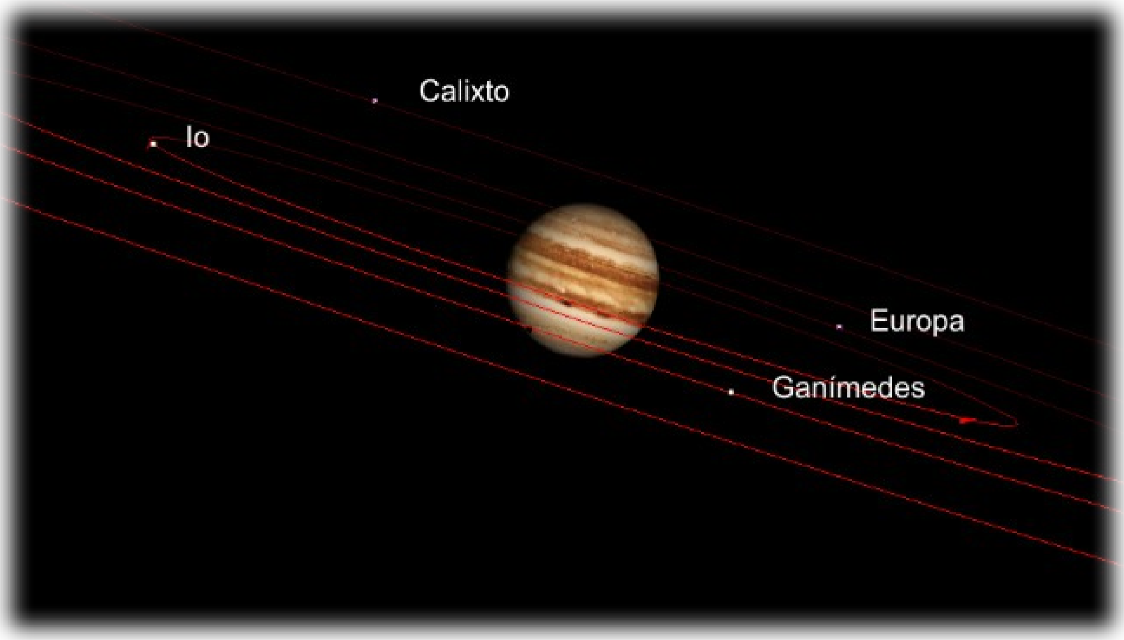
To obtain the mass of the Sun in ***kg***, we must put the other variables in I.S. units, i.e. the distance in meters and the period in seconds.

Substituting in the formula the values of a and ***P*** of our planet, the result for the mass of the Sun is: M_{Sun} = 1,989x10^{30} *kg*.

ACTIVITY 2

GALILEAN SATELLITES A MINIATURE SOLAR SYSTEM

In the following image we have identified the position of each of the Galilean satellites. It is important to ensure no mistake is made in the location of Io.



Io's orbit

The measurements obtained to calculate the distance from Io to the centre of the planet are:

- (336 +474) / 2 = 405 *pixels*
- 405 *pixels* x 1,036 *km*/*pixel* = 419,580 *km*

  A value very close to the actual one: 421,800 *km*

Jupiter's mass

To calculate the mass of Jupiter we will convert our values to the S.I.

- 421,800 *km* = 421,800,000 *m*
- 1.769 *days* = 1.769 x 24 x 60 x 60 = 152,842 s

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

Doing the calculations we get the mass of Jupiter, $M_J = 1.90 \times 10^{27}$ *kg*

Europa's orbital period

Measurements obtained to calculate the distance from Europa to the centre of the planet:

- (655 + 817) /2 = 736 *pixels*
- 736 *pixels* x 904,6 *km/pixel* = 665,786 *km*

A value close to the actual one: 671,400 *km*.

Europa's orbital period will then be as follows:

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

Clearing *P*:

$$P = \sqrt{\frac{4\pi^2}{G}\frac{a^3}{M}}$$

We get a period for Europe of: *P* = 303,176 *s* = 3.509 days

A value close to the actual one: 3.55 days

As an extension to this unit, students could be asked to calculate the mass of the Earth, using the distance from the Moon and its orbital period.

* The dark spot on the surface of Jupiter in image 668e000.hfit is simply the shadow of one of Jupiter's satellites on the planet's atmosphere. Specifically, it is the shadow of Ganymede.

For further information, visit our website: **www.iac.es/peter**

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