UNIT 5:

LAWS OF PLANETARY MOTION

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ACTIVITY 1

CALCULATION OF THE MASS OF THE SUN

OBJECTIVES

The following activity is intended for students to apply the laws of planetary motion and check Kepler's third law with the data provided for each planet. We will also calculate the mass of the star around which orbit all the planets of our Solar System.

EQUIPMENT AND MATERIAL

For the realization of this practice we are going to use a table of orbital data of the planets belonging to the Solar System. We will need a calculator to perform the calculations.

METHODOLOGY

Initially we will check if Kepler's 3rd law is satisfied for each planet using the known data of period and radius of their orbits.

Then we will calculate the mass of the Sun with the same data provided, using Newton's Law of Universal Gravitation.

PROCEDURE

The first thing we will do is to check Kepler's 3rd Law with the data of the orbital elements that we have in the following table. That is, we will check if it is observed:

$$\frac{P^2}{a^3} = Constant$$

where ***P*** is the period of revolution and ***a*** is the semi-major axis of the ellipse.

Planet	Period (years)	Semi-major axis (U.A)	P^2/a^3
Mercury	0,24	0,387	
Venus	0,62	0,723	
Earth	1,00	1,000	
Mars	1,88	1,524	
Jupiter	11,86	5,203	
Saturn	29,46	9,539	
Uranus	84,01	19,182	
Neptune	164,80	30,058	

Having verified the validity of Kepler's 3rd Law, let's try to estimate the mass of the Sun.

Using Kepler's 3rd Law and Newton's Law of Universal Gravitation we can obtain a formula that allows us to calculate the mass of the Sun with some simple, though tedious, calculations, since we are dealing with very large numbers. First, we must make some assumptions. One of them is that the orbits of the planets are circular, something we know is not true, because they are elliptical. Since they really are very close to a circle, for our purposes the approximation will not vary the result much.

Thus, the path that each planet makes around the Sun will be that of the length of a circle, that is, 2πa, where a is the average distance that separates the planet from the Sun.

We will omit how the formula is obtained. If you are curious, here are the two equations used, in addition to Kepler's 3rd Law above. Your teacher can explain how to proceed.

Law of Universal Gravitation: $F_G = G \frac{M_{Sun} m_{Planet}}{a^2}$

Centripetal force: $F_C = m_{Planet} \frac{v^2}{a}$

The formula we will finally use is:

$$M_{Sun} = \frac{4\pi^2}{G} \frac{a^3}{P^2}$$

To obtain the mass of the Sun in *kg* we must express the period ***P*** (time it takes for a given planet to orbit around the Sun) in seconds, and the distance ***a*** (average distance from the planet to the Sun) in meters. ***G*** is the universal gravitational constant and is 6.67-11 $N\ m^2\ kg^{-2}$.

This can be done with data from any planet but, as a start, let's try with data from the Earth. We know it takes 1 year to complete one orbit around the Sun. Since we have to express this number in seconds, then:

1 *year* = 365.25 *days* = 365.25 x 24 *hours* = 365.25 x 24 x 60 *minutes* = 365.25 x 24 x 60 x 60 x 60 *seconds*

Do this last calculation and you will have the period ***P*** of the Earth in seconds.

Now we will have to express the distance a in meters. We know that the Earth is at 1 *A.U.* (Astronomical Unit) from the Sun, which is equivalent to 149.6 million kilometers, then:

a = 1 *A.U.* = 149 600 000 *km* = 149 600 000 x 1000 *m*

Now, simply substitute all these values into the equation and, using a calculator (be careful with exponents), obtain the mass of the Sun in kg. What do you get? Try to write the quantity with all its zeros and you will see how big it is.

You can repeat the calculation with the data of ***P*** and a of any other planet (which you will find in the table above) and you will see that you get practically the same result. Remember to express ***P*** in seconds and ***a*** in meters. Don't you think it's amazing to be able to measure the mass of a body without even touching it?

For further information, visit our website: **www.iac.es/peter**

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